Archimedes' principle in fluidized granular systems

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We fluidize a granular bed in a rectangular container by injecting energy through the lateral walls with high-frequency sinusoidal horizontal vibrations. In this way, the bed is brought to a steady state with no convection. We measured buoyancy forces on light spheres immersed in the bed and found that they obey Archimedes' principle. The buoyancy forces decrease when we reduce the injected energy. By measuring ascension velocities as a function of Γ, we can evaluate the frictional drag of the bed; its exponential dependence agrees very well with previous findings. Rising times of the intruders ascending through the bed were also measured, they increase monotonically as we increase the density.

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Dry granular beds at rest are ubiquitous in our everyday life. Either held by boundaries (in silos, pharmaceutical containers, industrial barrels) or piled in open spaces, they stay motionless because their thermal energy is insignificant compared to their gravitational energy [1]. However, if an external force or a mechanical vibration is applied to them, a fascinating and rich dynamics, not observed in other phases of matter, might be triggered: avalanches [2], convection [3], compaction [4], jamming [5], segregation [6,7], pattern formation [8], buoyancy [9]. These granular phenomena are today, both experimentally and theoretically, intensively studied in many different types of systems and excitation conditions. A complete understanding of the above phenomena is needed not only for fundamental reasons but also due to the immense importance granulates have in industry. Furthermore, granular systems are simple models useful to understand phenomena observed at other scales, as crystallization and the glass transition in colloidal suspensions or jamming in spin glasses [10].

Nowadays, an important matter of concern is to find the limits and conditions within which granular dynamics might be described using well-established kinetic and hydrodynamics theories [11–14]. At the core of the discussions, there is the fact that due to the inelastic grain-grain and grain-wall collisions, vibrated granular systems are inherently out of equilibrium. Indeed, if a vertical mechanical vibration is the source of the energy injected to a granular system, the pumped energy is dissipated inside the bed through collisions and a kinetic energy gradient in the bed immediately develops [15], where the most commonly observed consequence is convection [3].

In the literature of granular physics there is not a clear distinction between flowing and fluidization. Normally, it is suggested that a granular system in ambient pressure becomes fluidized only when it flows (here, the term fluidization is used with the same caveat invoked by granular scientists when comparing a strongly vibrated granular medium with a standard liquid or a gas). However, we show in the present paper that a dry granular bed could be fluidized in such a way as to prevent any flowing. This can be achieved if the granular bed is put inside a "vibrational" bath that injects energy through the lateral walls of a container with vibrations perpendicular to gravity; in analogy with a standard fluid in a thermal bath which once in equilibrium presents no convection. Surprisingly, we found that a dry granular system brought to this state has hydrostatic properties.

The experimental setup consists of two concentric rectangular Plexiglas open boxes (see Fig. 1). The external box (35 cm per side) is sturdy. A 500-W loudspeaker is firmly mounted on each one of its vertical walls. The internal box ($10.5\ \text{cm} \times 10.5\ \text{cm} \times 20\ \text{cm}$) has moving lateral walls which are supported by eight beams attached to the loud speakers (two per speaker); the bottom of the box is fixed. In order to have independence in their movement, neighboring walls and bottom are separated by a small gap (≈ 1 mm). The speakers are electrically connected in parallel and fed with an amplified periodic voltage coming from a function generator HP-33120A. There are four electrical switches to individually turn on and off the speakers. By properly connecting these switches, walls can move in or out of phase. We always set the movement of opposite walls in opposite directions; in this context, and according to the schematic set up shown in Fig. 1, walls 1 and 3 move with excitations given by $x(t) = \pm A \sin(\omega t + \phi_1)$, while walls 2 and 4 (perpendicular to the plane of the page and therefore not shown) vibrate with displacements given by $y(t) = \pm A \sin(\omega t + \phi_2)$. The signs mean that opposite walls move in opposite directions.

A bi-dispersion consisting of glass beads with diameters of 3 and 4 mm was poured into the container (internal box); this bi-dispersion is meant to prevent any possible crystallization of the bed. When the loud speakers are turned on, the walls vibrate and the bed fluidizes. In order to maintain a constant bed volume, adjacent walls move out of phase ($\phi_2 - \phi_1 = \pi$). The maximum acceleration of the walls in our experiments was $\Gamma = A\omega^2/g = 10$, where g is the acceleration of gravity. The vibrations applied to the container are perpendicular to gravity, thus g is used here only as a reference. Naturally, due to the fact that the walls apply only horizontal impulses onto the granular bed and these are applied uniformly, from top to bottom, collisions between the beads do

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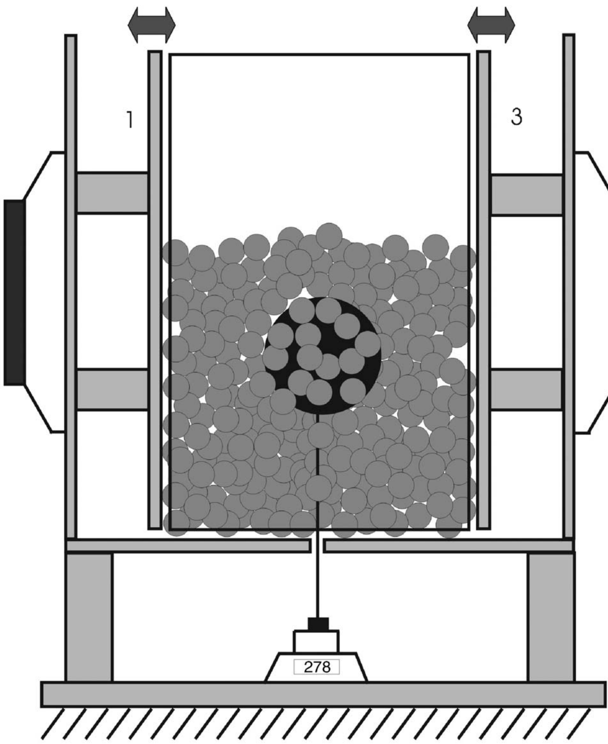

FIG. 1. Experimental setup. 500-W loudspeakers are firmly mounted on the lateral walls of an external and rigid box. The internal box, which acts as the bed container, has moving lateral walls. These are supported by eight beams attached to the loudspeakers (two per speaker). Walls and bottom do not touch each other.

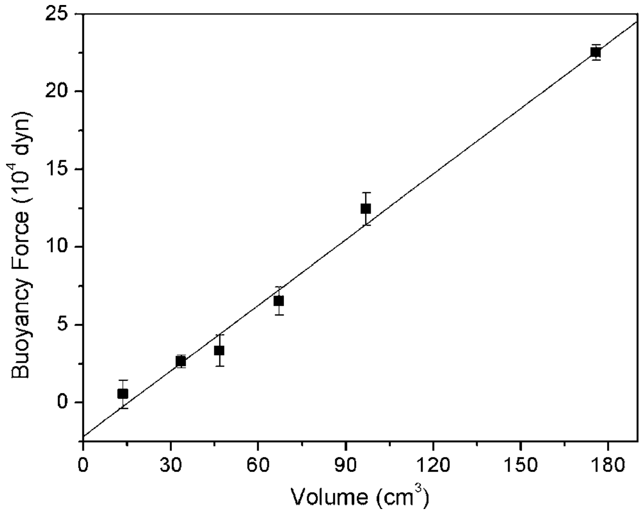

FIG. 2. Buoyancy forces for spheres of polystyrene (density 0.024 g/cc) plotted as a function of their volume for $\Gamma=9.5$ (linear frequency 50 Hz). The linear behavior indicates that buoyancies felt by immersed spheres in the granular bed are proportional to the volume they displace (Archimedes' principle). The slope of the line divided by g gives the effective density of the granular bed ($1.416\ \mathrm{g\ cm^{-3}}$).

not cause a net vertical transfer of kinetic energy. Also, the net drag between the walls and the particles is zero. Therefore convection cannot be triggered. Experimentally, we confirmed this prediction by putting tracer beads inside the bed: after several hours of vibrations, they barely move from their original positions.

The first property we would like to study in a granular bed fluidized with the previously described method is buoyancy. To this end, light polystyrene spheres (with density 0.024 g/cc) were buried into the material. There are two possible ways to do this: The first is to evacuate the container, put the sphere at the bottom and then pour the granulate back into the box. The second is to fluidize the bed (by turning on the walls) and then introduce the sphere until we touch with it the base of the container. Although a bit more time consuming, we prefer the first method. It is worth mentioning, however, the amazing experience one feels when introducing a sphere into the fluidized bed (second method), for it resembles the experience one feels when a light object, like an inflated ball, is introduced in water or any other liquid. Buoyancy forces were measured in the following way: One end of a fishing thread is first attached to a sphere, the other end is passed through a small hole located at the bottom of the container and then connected to a dynamometer (see Fig. 1). Thereafter, the container is filled with the granulate. At the beginning of the experiment, the thread is loose. When the vibrations are turned on, the bed immediately fluidizes and the buried sphere starts to ascend by buoyancy until the thread tension is big enough to maintain the sphere still. At that point, reached after several minutes, the lecture on the dynamometer gives the tension of the thread. The buoyant force is the sum of the measured tension plus the weight of the sphere. Buoyant forces for $\Gamma=9.5$ are plotted as a function of the spheres volume in Fig. 2. The resulting linear behavior is indicative that these forces obey Archimedes' principle, which states that a buoyant force felt by an object immersed in a fluid is proportional to the volume it displaces. The proportionality constant is the density of the displaced fluid times gravity. In our case, the slope of the line in Fig. 2, divided by g is (1.416 ± 0.06) g/cc, which must account for the effective density of the displaced granular fluid. The ratio between this effective density and the density of the glass beads (2.24 g/cc) is 0.63 ± 0.03. This value is in a close agreement with the volume fraction of a static random close packing phase for a hard sphere monodispersion: 0.64 [16]. We have carefully measured the volume fraction of the bi-dispersion used in our experiments (using a container with similar dimensions as the one we used to fluidize the bed) and found a volume fraction of 0.62. It is important to remark the fact that the fluidized bed barely expands and therefore its volume fraction, within the experimental error, is always the same. We observed that as the size of the intruder approaches the size of the beads, the intruder loses its identity and behaves just as any other bead in the granular medium. This behavior may explain why the fitting line in Fig. 2 does not go through the origin, suggesting that buoyancy does not show up until the volume of the intruder is considerably larger than the volume of the beads ($0.03\ \mathrm{cm^3}$).

Before describing further experiments carried out in the present study, a pertinent discussion follows. Buoyancy in a standard fluid arises from two facts: Pressure increases with

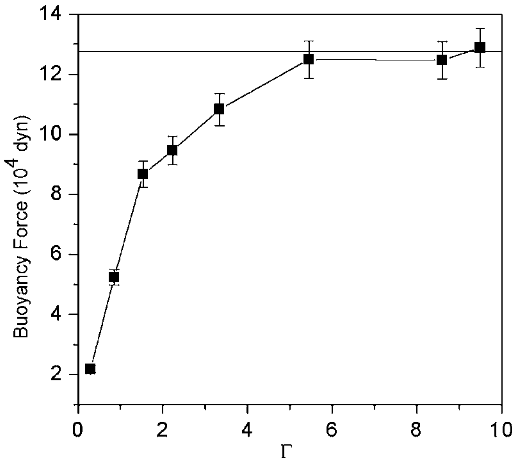

FIG. 3. Buoyancy force vs Γ for a 92-cm^3 sphere. The horizontal line corresponds to the force given by Archimedes' principle. At around $\Gamma \leqslant 5$, buoyancy forces drop.

depth and this is exerted in all directions (Pascal's principle). It is well known that static granular media do not follow this hydrostatic behavior due to the screening effect in the internal pressure caused by arches and force chains. However, in our experiments, the high frequency vibrations produced by the walls continuously break force chains and arches. Any possible pressure screening is therefore eliminated. It is interesting to mention that a cube made of polystyrene, put at the bottom of the container before pouring the beads into it, feels no buoyancy. But if the cube is buried into the bed when the system is already fluidized (second method), the cube rises as soon as it is released, since at least a layer of beads remains between the cube and the bottom of the box.

We now investigate how buoyancy forces change as we reduce the injected energy or temperature of the walls. In Fig. 3 we show buoyancy forces measured for a 92-cm^3 sphere as a function of Γ. The horizontal line is the buoyancy force given by Archimedes' principle (1.27×10^5 dyn), taking 1.416 g cm^{-3} as the effective density of the vibrated bed. Within the experimental error, buoyancy forces for $\Gamma \geqslant 5$ are similar to the theoretical value, while they deviate from this value for lower Γ's. Although this deviation could be considered as a breakdown of Archimedes' principle, we prefer to explain the deviation in the following terms: at low accelerations there are two phases coexisting inside the bed, a poorly fluidized phase in the center of the cell and a fluid close to the walls (i.e., only regions of the bed close to the walls are fully fluidized). Thus since part of the large 92-cm^3 sphere is embedded in the poorly fluidized phase of the bed and part is in the fluid, buoyancy forces drop substantially. The easiest way to probe this assumption is to use smaller intruders. We put two 8-cm^3 polystyrene spheres inside the bed, both buried to the same depth; one of them was put in the middle of the cell, the other at one of its corners. The latter, which is in the fully fluidized phase, ascends much faster.

The vertical position of the immersed 92-cm^3 sphere as a function of time, is plotted in Fig. 4 for three values of Γ. The positions were measured by filming the ascension of the fishing thread (previously detached from the dynamometer) against a fixed millimeter scale; we kept the thread vertical

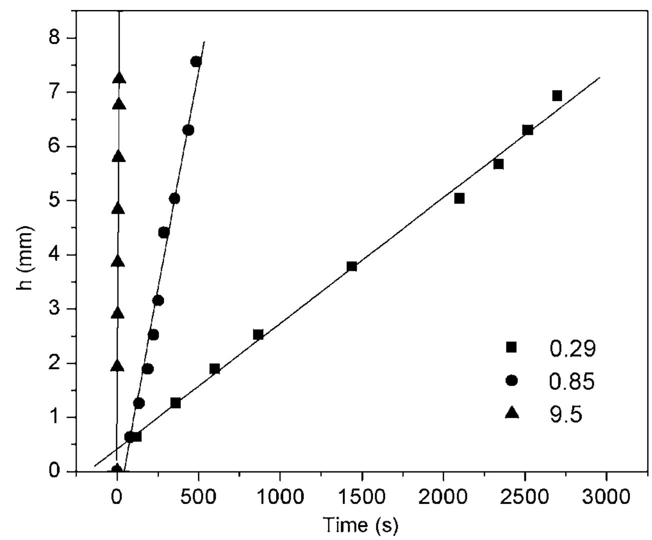

FIG. 4. Height time for some values of Γ. The lines show that the sphere moves with constant (terminal) velocities.

by hanging a small light clip to its free end. The obtained lines indicate constant ascension velocities, the slopes of such lines are the terminal velocities for each Γ.

In Fig. 5 we plot terminal velocities of the 92-cm^3 sphere as a function of Γ. Clearly, the velocities drop as the value of Γ is reduced. It is surprising that even if the walls vibrate at accelerations much lower than gravity, buoyancy persists. For instance, for $\Gamma = 0.29$ the bed looks like an unperturbed solid in a first sight (one has to pay a special attention to discover a minute trembling of the beads), yet, buoyancy is not zero and the intruder ascends slowly to the top. At $\Gamma = 9.5$ the 92-cm^3 sphere takes 5 sec to ascend 7 mm, while at $\Gamma = 0.29$ the same sphere takes around 2700 sec!

Some authors have given some clues about the mobility of objects in static or perturbed granular media [20–22]. In our case, under the reasonable hypothesis that the drag force felt by a buoyant sphere ascending through the bed is proportional to its velocity ($F = \zeta v$, see the work of Zik *et al.*

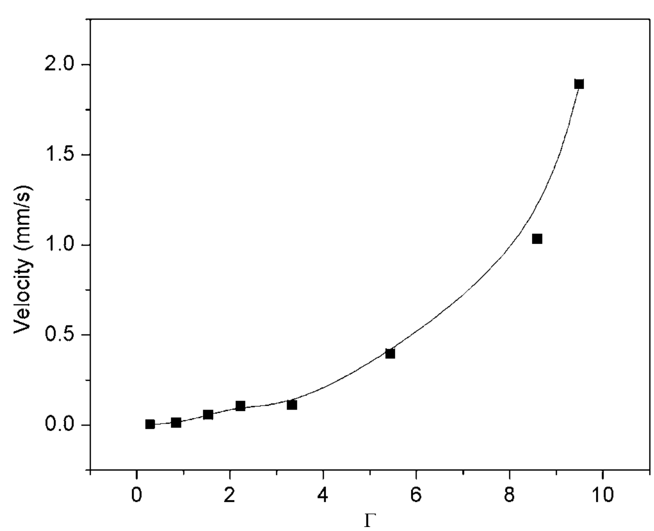

FIG. 5. Ascension velocities for the 92-cm^3 sphere vs Γ. The solid line is only a guide to the eye.

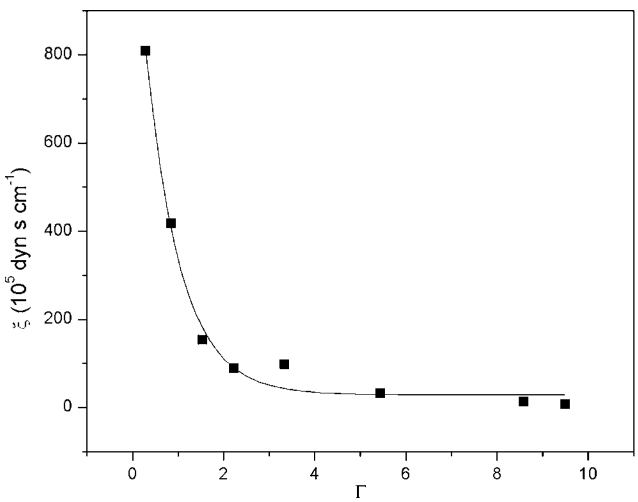

FIG. 6. ζ vs Γ. The exponential solid line is only a guide to the eye.

[21], for instance), we can easily estimate the friction coefficient ζ [23]. In Fig. 6, this is plotted as a function of Γ. The exponential dependence of ζ with Γ agrees very well with the previous findings of Zik *et al.* [21] for a small bead dragged horizontally through a vertically vibrated granular system.

A classical phenomenon observed in granular physics is segregation, where the so called Brazil nut problem (BNP) stands as its emblematic object of study [7,17,18]. The BNP, intensively studied since its discovery in 1939 [19], is related to the ascension of an intruder in a granular bed subjected to vertical vibrations. Since the BNP has never been observed in granular systems shaken with horizontal vibrations, we would like to address this important problem in the context of the present study. One finds in the literature that the relevant parameter in the BNP is the rising time of the intruder T_R. Therefore rising times of ascending spheres through our fluidized bed were measured in our experiments. Figure 7 shows these rising times plotted as a function of the relative density ρ_r (the density of the polystyrene spheres divided by the density of the glass beads) for various diameters. The

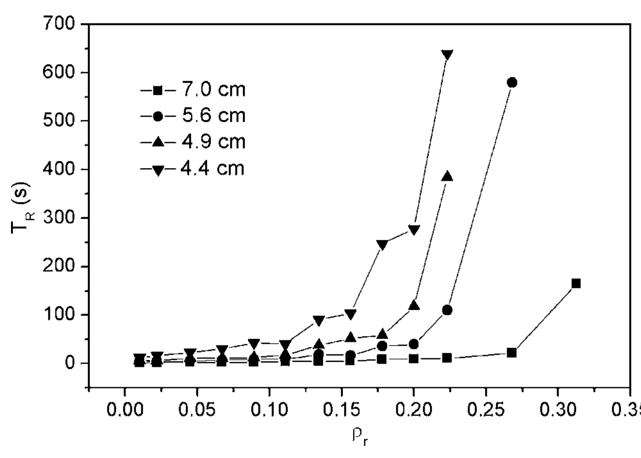

FIG. 7. Rising times of the spheres in the fluidized bed plotted as a function of relative density for different diameters.

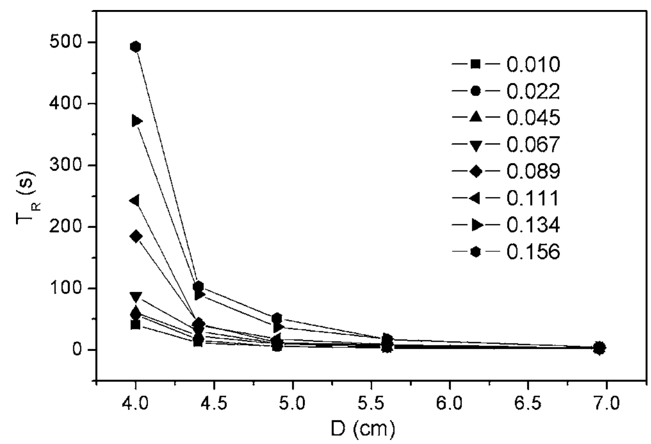

FIG. 8. Rising times plotted as a function of diameter for different relative densities.

spheres were buried at different positions, such that they always ascend the same distance (2.2 cm). The density of the spheres was changed by doping the spheres with steel beads of different sizes. We note that rising times rapidly increase with ρ_r, the effect is even more pronounced for smaller diameters. It is important to remark that this monotonic rising behavior is not in contradiction with the most recently published works in the BNP, where a nonmonotonic dependence in real systems (in air) is observed as the density is changed [7,18]. The reason is simple: it has been demonstrated that the nonmonotonic trend is produced by the competition of two different mechanisms: inertia and convection [6,7]. In the experiments reported here there is no convection, furthermore, inertia is not present because the vibrations are horizontally injected. With the the absence of these two mechanisms, buoyancy stands as the only driving force behind segregation. We have also confirmed that in this fluidization regime, objects whose densities are much larger than the effective density of the bed (1.416 g/cc), sink. Objects of density just above the effective density of the bed were expected to sink, but we have observed they do not. Glass spheres (2.5 g/cc), for instance, do not sink and they rise if they are immersed in the glass bed. Furthermore, steel spheres (7.86 g/cc) sink but they present two regimes in the process of sinking; for instance, a steel sphere of 1.9 cm of diameter can sink two or three centimeters in several seconds, and then it slowly continues its sinking taking more than 30 min to advance the same distance again. To explain this behavior, we recall that an object immersed in a granular medium induces the formation of arches which prevents the object from sinking. It seems that when the density of the intruder is similar or larger to the effective density of the bed, the fluidization mechanism (vibrating walls) fails to break the arches that are formed as a reaction against the sinking of the intruder. Further research to understand the process for relative densities close to or larger than one is warranted. Figure 8 depicts rising times as a function of diameter D for various relative densities. The well known paradigm of granular segregation, establishing that larger intruders segregate faster than smaller ones with the same density, is also observed in our fluidized conditions. Nevertheless, in a vertically shaken granular bed this happens due to convection [7], but in the present experiments, the paradigm is fulfilled due to buoyancy.

Summarizing: for the first time a granular bed has been perturbed by injecting energy through the walls of a container, in such a way that we are able to fluidize the system creating a steady state with no convection. We have measured buoyancy forces on light spheres immersed in the fluidized bed and found that these forces are proportional to the displaced volume, according to Archimedes' principle. Further, these hydrostatic forces drop as the bed loses its full fluidization. Also, rising times of ascending spheres through this liquidlike system were measured as a function of relative density ρ_r and size. A monotonic increasing and decreasing behavior, respectively, was observed. Our findings might open new fronts of research to understand the behavior of fluidized granular beds in a steady state, and uncover a new mechanism to pursue granular segregation that might have important applications in the industry.

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